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A proposed mathematical model for the resolution of the Home Health Care (HHC) Nurse Scheduling Problem for Peritoneal Dialysis (PD) patients

Un modèle mathématique proposé pour la résolution du problème de planification des infirmiers de Soins à Domicile (SAD) pour les patients sous Dialyse Péritonéale (DP)

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Abstract

Home HealthCare (HHC) are crucial for Peritoneal Dialysis (PD), ensuring patient's satisfaction while minimizing both patient waiting times and the risks related to patients travel to hospitals. However, research in this area remains under-explored in Tunisia. This article introduces a mathematical model for nurse assignment and patients scheduling, aiming to optimize HHC for patients using PD in Tunisia. The proposed model aims to minimize the total start time of service for patients, thus demonstrating its applicability in the Tunisian context. To solve our model, we resort to two random instances since real data is not accessible. The results obtained highlight the model's effectiveness in improving the quality of care and patient satisfaction, while also reducing costs and the risks associated with hospital travel.

Keywords : Home Health Care (HCC); Peritoneal Dialysis (PD); Mathematical Model; Nurse Assignment; Patient Scheduling**.**

Résumé

Les Soins Á Domicile (SAD) sont cruciaux pour la Dialyse Péritonéale (DP), en assurant la satisfaction des patients afin de minimiser à la fois les temps d'attente et les risques associés aux déplacements vers les hôpitaux. Cependant, la recherche dans ce domaine reste peu explorée en Tunisie. Cet article introduit un modèle mathématique pour l'affectation des infirmiers et la planification des patients, visant à optimiser les SAD pour les patients utilisant la technique de DP en Tunisie. Le modèle proposé minimise le temps total de début du service pour les patients, démontrant ainsi son applicabilité dans le contexte Tunisien. . Pour résoudre notre modèle, nous avons recourt à deux instances aléatoires puisque les données réelles ne sont pas accessibles. Les résultats obtenus soulignent l'efficacité du modèle pour améliorer la qualité des soins et la satisfaction des patients, tout en réduisant les coûts et les risques associés aux déplacements hospitaliers.

Mots clés : Soins à Domicile (SAD) ; Dialyse Péritonéale (DP) ; Modèle Mathématique ; Affectation des Infirmiers ; Planification des Patients.

Introduction

In recent decades, and particularly following the of COVID-19 pandemic which upended the healthcare landscape by complicating the process of treatment of chronically ill patients. Home Health Care (HHC) has proved its efficiency and effectiveness in critical situations to save humans life.

The goal of HHC is to facilitate patients in receiving necessary care within the comfort and familiarity of their own homes, while preserving a higher level of independence and quality of life. According to (Eveborn et al., 2006) and (Frifita et al., 2017), HHC has been suggested as a cost-effective alternative for delivering medical assistance to the public. It is structured to utilize the existing hospital bed availability while ensuring the comfort and hygiene of patients received care at home (i.e avoiding the risk of hospital-acquired infectious).

As the global burden of chronic kidney disease continues to increase, so does the need for a cost-effective renal replacement therapy. Providing optimal medical care is a significant challenge, especially for end-stage renal disease (ESRD) patients undergoing inpatient dialysis (Samarra Badrouchi et al. 2022).

Dialysis is a life-prolonging treatment for patients with end-stage renal disease. It has two basic types of dialysis concerning hemodialysis and peritoneal dialysis respectively. In the first one, blood is purified directly by an extracorporeal dialysis machine, whereas in the second one (peritoneal dialysis), the peritoneal cavity is filled with sterile fluid and the peritoneum acts as a natural filter. Peritoneal dialysis can be done by patients at home, whereas hemodialysis is usually done by a trained medical professional in a dialysis center or hospital. Although the clinical outcomes of both dialysis methods are comparable, peritoneal dialysis offers the greatest potential for more flexible treatment schedules due to its enhancement of patient comfort and its cost effectiveness (Jaar et al., 2005; Weinhandl et al., 2010; Mehrotra et al., 2011). According to Habib et al., 2017, the primary factor influencing the selection of PD is the ability to perform the treatment at home. Nevertheless, despite its advantages, the widespread adoption of home dialysis is hindered by the challenges faced by some elderly and frail patients to self-medicate (Berger et al., 2009; Karopadi et al., 2013; Wong et al., 2016). Specifically, peritonitis is the leading cause of technique failure, accounting for 30.1% of patients being permanently transitioned to hemodialysis De Moraes TP et al. (2014). In our cohort, peritonitis was the sole independent predictor of technique failure. However, other studies have identified additional independent predictive factors, including age, center experience and the use of automated peritoneal dialysis as initial PD modality (De Moraes TP et al. 2014; Lim WH et al. 2011).

Samarra Badrouchi et al. (2022) also confirm that Peritoneal Dialysis (PD) is an excellent choice for Renal Replacement Therapy (RRT) during the COVID-19 pandemic. This is because many aspects of PD can be managed remotely, reducing the need of routine hospital visits and helping to maintain social distancing- a crucial measure for breaking the transmission chain of COVID-19.

Having provided a proper set of services for patients with end-stage renal disease at home, PD appears to be an ideal option. Treating PD patients at home falls under the realm of Home Health Care (HHC). By developing an appropriate model based on special needs of PD patients and optimizing the function of this model, it could potentially replace HD. Ultimately, this optimized model could be effectively implemented by HHC companies in real-world settings.

conducted a methodological study aimed to evaluate home dialysis, specifically focus on the Tunisian context. Their findings indicate that Home Peritoneal Dialysis has the lowest annual cost per patient compared to the two other modalities examined: Conventional and Short-Daily Home Hemo-Dialysis (HHD). To the best of our knowledge, this study represents the first of it kinds to address this issue in North and Sub-Saharan Africa.

This article seeks to address the following problem: How to optimize patient scheduling for peritoneal dialysis within the context of Home Health Care in Tunisian context?

In the Tunisian context, HHC has not yet been implemented for PD. To enhance the quality of healthcare and boost patient satisfaction in Tunisia, and to address the ongoing challenges faced by patients opting for PD- particularly those with limited knowledge about this dialysis method- we propose the adoption of HHC for PD in this paper. We present a mathematical model designed for the planning of HHD with the Tunisian context.

The proposed methodology involves the development of a mathematical model for optimizing nurse assignment and patient scheduling in HHC for PD patient in Tunisia, aiming to minimize total service start times and enhance patient satisfaction and care quality. To test the applicability of the presented model, we have used two random instances.

This article is structured into three main sections. The first section presents a literature review, examining existing studies and models related to Home Care Services (HCS) for Peritoneal Dialysis (PD). This review contextualizes the current state of research and identifies gaps, particularly in the Tunisian context. The second section introduces the proposed mathematical model for nurse assignment and patient scheduling, detailing the assumptions, variables, and objectives of the model. Finally, the third section presents the implementation and computational results, demonstrating the applicability and effectiveness of the proposed model in the Tunisian context.

1. Literature review

The interest in the HHC research field is growing continuously. This problem can be seen as an Operations Research point of view (Begur et al., 1997; Cheng and Rich, 1998). In this section, we focus on recent advancements and insights from scientific articles that have addressed the complexities of HHC problems.

Focusing on the integration of nurse scheduling, routing, and patient assignments, we explore how contemporary studies have utilized optimization techniques to enhance operational efficiency and patient care quality. These key contributions highlight the application of mixed integer programming, meta-heuristics, and multi-objective approaches, providing a foundation for understanding the current state of research.

In Euchi et al. (2020), the problem is described as a VRSP (Vehicle ReScheduling Problem) with time windows with synchronization of visits, exclusively based on an artificial intelligence technique to optimize the offered services within a distributed environment. The authors' approach is grounded in the model developed by (Kandakoglu et al., 2020). In this respect, they use a two-phase approach. Initially, they implement a k-mean clustering algorithm (CA) to identify several caregiver routes. Subsequently, they apply a hybrid approach combining an ant colony system (ACS) with clustering (hybrid ACS-CA) to optimize the problem in a distributed manner. The proposed algorithm is tested on datasets generated by (Bredström and Rönnqvist, 2008) demonstrating the ability to achieve optimal or near-optimal solutions within a reasonable timeframe.

Nasir and Kuo (2020) focus on the elderly population by developing a decision support framework to address the Home Healthcare Routing and Scheduling Problem (HHCRSP), enabling the concurrent development of schedules and route plans for HHC staff and Home Delivery Vehicles (HDVs). A Mixed Integer Linear Programming (MILP) model is developed to characterize the optimization problem. To tackle large-scale instances, they propose a hybrid genetic algorithm for home care logistics planning, incorporating synchronized visits and multiple nurses. The computational tests affirm that the proposed method is significantly shorter computation time compared to MIP model.

Demirbilek et al. (2021) treat the Home Health Care Routing and Scheduling Problem (HHCRSP) of multiple nurses in a dynamic environment with both Soft and Hard Time Windows (HTWs) associated with caregivers and patients, respectively. The authors take all nurses and future demand into account at the same time during the decision process. In this context, they propose a scenario-based method for several nurses to determine the maximum number of patient' visits for a group of nurses during the planning horizon. To solve their problem, a meta-heuristic method based on the Iterated Local Search (ILS) algorithm is adopted. The model is tested with instances based on real-world data. The proposed solution aims to minimize the system's total cost while maximizing the total satisfaction of patient's preference.

Euchi and Sadok (2021) introduce a new variant of Vehicle Routing Problem, the VRP with drones (VRPD) in according to the need of routing a heterogeneous fleet of drones and trucks from large areas. The specificity of this variant is that trucks and drones can deliver parcels independently or cooperatively. They formulate the problem as a MILP. Their research contribution lies in devising a resolution method from operational research to optimize personal tours for visiting patients' homes, aiming to decrease caregivers' travel time and enhance service quality. To solve this problem, they propose a new hybrid algorithm that combines the Ant Colony System with a Genetic Algorithm (CS – GA. The proposed solution methodology is implemented based on a set of instances from the literature (Bredström and Rönqvist, 2008; Decerle et al., 2018) demonstrating a strong balance between efficiency and reliability.

Nikzad et al. (2021) introduce the Stochastic Districting-staff Dimensioning-Assignment-Routing Problem (SDDARP) and model it using a Mixed Integer Linear Programming (MILP) framework. They propose a two-stage approach for resource planning in Home Health Care (HHC) problems: districting and staff dimensioning in the first stage, followed by routing and assignment in the second stage. To implement this, they develop a variant of the progressive Frank-Wolfe and Hedging algorithms. A metaheuristic procedure is devised to minimize travel costs. The solution methodology is validated using instances from Fikar and Hirsch (2015), demonstrating the ability to achieve optimal or near-optimal solutions within a reasonable computational timeframe.

Li et al. (2021) introduce in their work a new variant VRSP integrating outpatient services, and considering time windows, skills, and working regulations as constraints. The problem is formulated as a Mixed-Integer, Non-Linear (MINonLP) and a convex programming model, incorporating travelling costs, waiting time, and patients' preference objective functions, is developed. To solve large-scale problems, authors propose a Hybrid Genetic Algorithm (HGA) with outer-approximation method. No benchmark exists for the problem, for this reason, the authors generate instances based on Solomon’s VRPTW benchmark (Solomon, 1987) and combine the characteristics of the realistic problem. The computational experimentations show that the solution can be considered as a high quality solution with regards to reasonable computing time, and can solve a set of much large instances than the exact method.

Shahnejat-Bushehri et al. (2021) introduce a robust optimization model for the Home Health Care Routing-Scheduling Problem (HHCRSP) considering uncertain services and travel times. They formulate a Mixed Integer Linear Programming (MILP) model and solve it using a combination of Monte Carlo simulation and metaheuristic algorithms, including Simulated Annealing (SA), Genetic Algorithm (GA), and Memetic Algorithm (MA). The model's validation is based on Solomon's Vehicle Routing Problem with Time Windows (VRPTW) benchmark (Solomon, 1987). The results confirm that the proposed method is efficient, providing near-optimal solutions and acceptable computational times.

Malagodi et al. (2021) tackle the short-term and very-short-term planning challenges for a novel and intricate class of healthcare services, drawing inspiration from a real-world case study in the USA. They formalize the healthcare assignment and scheduling problem, incorporating chargeable overtime and both strict and soft preferences, and propose a Mixed Integer Linear Programming (MILP) model to address it. The objectives are to minimize non-chargeable overtime expenses for the provider, reduce the count of unmet preferences (differentiating between strict and soft), and decrease the total travel time for caregivers.

Kordi et al. (2023) present in their research a multi-objective mixed-integer programming model for optimizing Home Health Care (HHC) services, addressing the inefficiencies of manual nurse planning. The model integrates financial, environmental, workload balance, and service quality objectives. It make into consideration several nurse skill levels, patient preferences, and vehicle types. The epsilon-constraint method and a Multi-Objective Variable Neighborhood Search (MOVNS) are employed to solve instances of different sizes (small and large instances). A sensitivity analysis and a real-world case study validate the model's effectiveness in enhancing HHC service planning.

Alkaabneh et al. (2023) tackle, in their work, the inefficiencies in home care service scheduling by proposing a mixed integer programming model that holistically addresses nurse-patient assignments, work schedules, and routing. The proposed model is designed as a multi-objective problem, seeking to reduce costs and improve nurse-patient compatibility. They introduce a branch-and-price algorithm and a two-stage meta-heuristic to solve the problem, evaluating their performance against sequential and conventional solver approaches. Computational experiments demonstrate that their integrated model can lead to cost savings and better compatibility, with no trade-off between the two objectives. The developed methods prove more effective than traditional solvers, underscoring the advantages of comprehensive optimization in home care planning.

Nasir et al. (2024) have addressed the challenges of meeting high demand in HHC supply chain, exacerbated by vehicles shortages and disruptions like pandemics and natural disasters. They proposed multi-depot, multi-period optimization model with precedence constraints considering uncertain demand quantities for medical supplies and dynamically changing patient priorities. Due to a unique three-phase solution approach integrated with stochastic simulation, authors have determined optimal base locations for Mobile Health Facilities (MHFs) and fleet sizes for HHC vehicles. To evaluate the performance, they are based on realistic data related to HHC service in Hong Kong.

As a conclusion, each of these papers contributes to the field of HHC optimization by proposing novel models, algorithms, and approaches that aim to enhance operational efficiency, reduce costs, and improve patient care quality. The methodologies range from mathematical programming and metaheuristics to robust optimization and hybrid algorithms, demonstrating the diverse strategies employed to tackle the complexities of HHC problems.

The studied literature review is summarized in tables 1 and 2.

Table N°1: A comparative table of advanced works related to HHC

Authors	Problem description	Model/Technique	Key contributions	Validation/Testing
Euchi et al. (2020)	VRSP with time windows and synchronization of visits	Two-phase approach: k-mean clustering (CA) followed by hybrid ACS-CA	Achieves optimal or near-optimal solutions within reasonable timeframe.	Datasets from Bredström and Rönnqvist, 2008
Nasir and Kuo (2020)	HHCRSP with concurrent scheduling and routing for HHC staff and HDVs	MILP model with a hybrid genetic algorithm	Significantly shorter computation time compared to MIP model; incorporates synchronized visits and multiple nurses	Not specified
Malagodi et al. (2021)	Healthcare assignment and scheduling problem with overtime and preferences	MILP model	Minimizes non-chargeable overtime and travel time	Real-world case study in the USA
Demirbilek et al. (2021)	HHCRSP with dynamic environment and soft/hard time windows	Iterated Local Search (ILS) meta-heuristic	Maximizes patient visits and satisfaction	Real-world data
Euchi and Sadok (2021)	VRPD with heterogeneous fleet of drones and trucks	MILP model with a hybrid Ant Colony System and Genetic Algorithm (CS – GA)	Optimizes personal tours for visiting patients' homes.	Instances from Bredström and Rönqvist, 2008; Decerle et al., 2018
Nikzad et al. (2021)	SDDARP in HHC	MILP framework with a variant of the progressive Frank-Wolfe and Hedging algorithms	Minimizes travel costs	Instances from Fikar and Hirsch, 2015
Li et al. (2021)	VRSP integrating outpatient services with constraints	MINonLP model with a Hybrid Genetic Algorithm (HGA) with outer-approximation method	Solves large scale problems	Generated instances based on Solomon, 1987
Shahnejat-Bushehri et al. (2021)	HHCRSP with uncertain services and travel times	MILP model with Monte Carlo simulation and metaheuristic algorithms	Provides near-optimal solutions	Solomon's Vehicle Routing Problem with Time Windows (VRPTW) benchmark
Kordi et al, (2023)	Multi-objective optimization of HHC services	Multi-Objective Mixed-Integer Programming model with	Enhances HHC service planning; validated with sensitivity analysis	Real-world case study

		epsilon-constraint and MOVNS		
Alkaabneh et al. (2023)	Integrated nurse-patient assignments, work schedules, and routing	Mixed Integer Programming model with branch-and-price algorithm and two-stage meta-heuristic	Leads to cost savings and better compatibility	Not specified
Nasir et al. (2024)	Multi-depot, multi-period optimization with uncertain demand and dynamic priorities	Multi-depot, multi-period optimization model with stochastic simulation	Determines optimal base locations and fleet sizes for HHC vehicles	Realistic data related to HHC service in Hong Kong

Source: Authors

Table 1 summarizes various research studies from 2020 to 2024.

Following is an analysis of the key elements from the first table.

- Problem description: we have observed that the problems range from basic vehicle routing with time windows and synchronization constraints to more complex scenarios involving dynamic environments, uncertain service, travel times and the integration of outpatient services. The challenges are specifically adapted to address the unique requirements of HHC, emphasizing the importance of accommodating patient preferences, maintaining high service standards and optimizing operational effectiveness.

- Model/technique: the set models and techniques (MILP, ILS, ACS, GA, Monte Carlo,…) reflect the complexity of the problem. This diversity in techniques underscores the multidisciplinary and complex nature of optimizing HHC logistics.

Table N°2: Set of objectives and constraints taken into consideration in the studied papers

	Objectives							Constraints					
	T T	***TC***	***TD***	***WT***	***OT***	***Pref***	***Num Nurs***	***BW***	***TW***	***SR***	***WR***	***Breaks***	***OT***
Euchi and al (2020)		*							*	*	*		
Nasir and Kuo (2020)	*	*							*	*	*		
Demirbilek et al. (2021)		*	*			*	*	*	*		*		
Euchi and Sadok (2021)	*					*	*	*	*		*		
Nikzad et al. (2021)		*				*	*		*	*	*		
Li et al (2021)	*			*		*			*	*	*		
Shahnejat-Bushehri et al. (2021)	*	*						*	*	*			
Malagodi et al. (2021)	*	*			*	*		*	*	*	*		*
Kordi et al. (2023)		*				*	*	*		*			
Alkaabneh et al. (2023)		*				*	*			*			
Nasir et al. (2024)		*				*	*	*		*	*		

Source: Authors

Based on the table (2), the most frequently addressed objective seems to be "Travel Time", as it is marked in several articles (Euchi et al. (2020), Nasir and Kuo (2020), Shahnejat-Bushehri et al. (2021)). The most commonly considered constraints is "Skill-requirements", which is highlighted in various papers (Nasir and Kuo (2020), Demirbelik et al. (2021), Malagodi et al. (2021), Kordi et al. (2023), Nasir et al. (2024)). On the other hand, "Wait Time" and "Breaks" are less frequently.

The literature review conducted within this research study reveals several limitations pertaining to HHC management. Notably, despite the significance of this issue, research in this domain is not yet widespread. Certain constraints, such as periodic maintenance of dialysis machines and the routine visits by nurses to conduct analyses, have not, to our knowledge, been widely integrated into the existing literature. Furthermore, only one study (Safa Chabouh et al., 2022) has specifically addressed HHC management in Tunisia, indicating that this approach has yet to be fully adopted in Tunisia.

In Tunisia, the majority of patients requiring dialysis are limited to hemodialysis due to their lack of familiarity with peritoneal dialysis. Despite their urgency of their condition, these patients often endure prolonged waiting times at hospitals. Additionally, some patients are compelled to perform peritoneal dialysis independently without adequate oversight. They must also undertake the burden of travelling to hospitals to obtain the requisite medications for their dialysis treatments, a practice that may exacerbate their health issues.

To ameliorate the quality of healthcare in Tunisia and improve the patients' satisfaction, we propose to require to HHC for peritoneal dialysis. In this paper, we present a mathematical model for the scheduling of nurses for peritoneal Dialysis patients in order to accelerate the total access time to each patient to do its healthcare at the corresponding time without delays.

2. Proposed Mathematical Model

The proposed mathematical model is formalized with a focus on the Tunisian context. It incorporates two innovative constraints: the frequency of nurse visits for the maintenance of the cycler and the frequency of the nurses visits for conducting analyzes on patients.

2.1. Sets

N: Set of nurses

P: Set of patients

T: Set of shifts

$W = N \cup P$: Set of locations

2.2 Indices

i: Index of patient

t: Index of shift

n: Index of nurse

2.3 Input parameters

TT_{ij}:Travel time from location $i \in W$ to location $j \in W$

$$V_{it} = \begin{cases} 1, if\ patient\ i\ must\ be\ visited\ in\ shift\ t \\ 0, otherwise \end{cases}, i \in P, t \in T$$

ST_j^t: Service time of patient $j \in P$ at shift $t \in T$

a_{it}: Earliest allowed service start time of patient $i \in P$ at shift $t \in T$

b_i: Earliest allowed service start time of patient $i \in P$ shift $t \in T$

T_{max}: Maximum service time

2.4 Decision variables

$$X_{ij}^{nt} = \begin{cases} 1, \quad if\ nurse\ n\ goes\ from\ location\ i \in W\ to\ location\ j \in W\ at\ shift\ t \\ 0, otherwise \end{cases}$$

S_i^{nt}: Service start time of a patient i visited by a nurse n at the shift t, $i \in P, n \in N, t \in T$

2.5 Objective function

The objective of this study consists on the minimization of the total access time to the different patients (see Equation 1):

$$Min \sum_{i=1}^{W} \sum_{j=1}^{W} \sum_{n=1}^{N} \sum_{t=1}^{T} (TT_{ij} + ST_j^t) X_{ij}^{nt} + \sum_{i=1}^{P} \sum_{n=1}^{N} \sum_{t=1}^{T} S_i^{nt} \quad (1)$$

2.6 Constraints

The various constraints that must be taken into account and respected to model our problem are summarized as follows:

Constraint (2) ensures that each patient must be visited by only one nurse at a shift:

$$\sum_{n=1}^{N} \sum_{j=1}^{W} X_{ij}^{nt} = V_{it}, \forall\ i \in P, t \in T \quad (2)$$

Constraint (3) guarantees that each nurse visiting a location must leave it:

$$\sum_{n=1}^{N} \sum_{j=1}^{W} X_{ji}^{nt} = \sum_{n=1}^{N} \sum_{j=1}^{W} X_{ij}^{nt}, \forall\ i \in W, t \in T \quad (3)$$

Constraint (4) makes sure that each nurse must start its workday from home and finish its workday at home:

$$\sum_{j=1}^{P} X_{nj}^{nt} = \sum_{i=1}^{P} X_{in}^{nt} \ ,\forall\ n \in N, t \in T \quad (4)$$

Constraint (5) ensures that a nurse cannot visit another nurse:

$$X_{nn}^{nt} = 0 \ ,\forall\ n \in N, t \in T \quad (5)$$

In constraint (6), a nurse cannot return to a visited patient in a routing tour:

$$\sum_{n=1}^{N} X_{p1p2}^{nt} + \sum_{n=1}^{N} X_{p2p1}^{nt} \leq 1, \forall\ p1 \in P, p2 \in P, t \in T \quad (6)$$

Constraint (7) ensures that at a shift, each nurse can begin its routing tour by no more than one patient:

$$\sum_{i=1}^{P} X_{ni}^{nt} \leq 1, \forall\ n \in N, t \in T \quad (7)$$

Constraint (8) guarantees that at a shift, each nurse must return to its home after its healthcare routing:

$$\sum_{i=1}^{P} X_{in}^{nt} \leq 1, \forall\ n \in N, t \in T \quad (8)$$

In constraint (9), each patient is visited by only one nurse at a shift:

$$\sum_{j=1}^{W} X_{ij}^{nt} = \sum_{j=1}^{W} X_{ji}^{nt} \ ,\forall\ i \in W, n \in N, t \in T \quad (9)$$

Constraint (10) ensures that each patient should be visited and served by the same nurse at a shift:

$$X_{ij}^{nt} = 0, \forall\ i \in W, j \in W, n \in N, t \in T, i \in N, i! = n \quad (10)$$

Constraints (11) and (12) guarantee that the services start time increases all along the routing of a nurse going from a patient to another. We can note here that the service start time of a destination patient is composed of the service start time of the origin patient, the service duration and the travel time:

$$TT_{ij}\, X_{ij}^{nt} \leq\ S_j^{nt} + M\left(1 - X_{ij}^{nt}\right), \forall\ i\ \in N, j\ \in P, n \in N, t \in T \quad (11)$$

$$S_i^{nt} + ST_i + TT_{ij} \leq\ S_j^{nt} + M\left(1 - X_{ij}^{nt}\right), \forall\ i\ \in P, j\ \in P, n \in N, t \in T \quad (12)$$

In constraint (13), the service start time of a patient served by a given nurse increases only if this patient is visited by the corresponding nurse:

$$S_j^{nt} \geq X_{ij}^{nt}, \forall\ i\ \in W, j \in P, n \in N, t \in T \quad (13)$$

Constraint (14) ensures the patient visits start within the corresponding earliest and latest start times:

$$a_i \sum_{j=1}^{W} X_{ij}^{nt} \leq S_i^{nt} \leq b_i \sum_{j=1}^{W} X_{ji}^{nt}, \forall\, i \in P, n \in N, t \in T \quad (14)$$

Constraint (15) guarantees that each nurse must perform his/her tour within the shift duration:

$$\sum_{i=1}^{P} \sum_{j=1}^{P} (S_i^{nt} + (ST_i^t + TT_{in})X_{ij}^{nt}) \leq T_{max}, \forall\, i \in P, n \in N, t \in T \quad (15)$$

Constraint (16) ensures that after 7 shifts, the patient must be visited by a nurse for the cycler maintenance:

$$\sum_{j=1}^{P} \sum_{n=1}^{N} X_{ij}^{nt} \geq 1, \forall\, i \in P, t \in T, t = R * 7, R = [1 \dots 90] \quad (16)$$

Constraint (17) ensures that after two months, the patient must be visited by a nurse to do analyze:

$$\sum_{j=1}^{P} \sum_{n=1}^{N} X_{ij}^{nt} \geq 1, \forall\, i \in P, t \in T, Day = R * 60, R = [1 \dots 90], Day = 1 \ if \ t \leq 3, \frac{t}{3} \ if \ t\%3 = 0 \ and \ \left(\frac{t}{3} + 1\right) if \ t\%3! = 0$$

(17)

Constraint (18) guarantees that the routing decision variables are of binary type:

$$X_{ij}^{nt} \in \{0,1\}, \forall\, i, j \in W, n \in N, t \in T \quad (18)$$

Constraint (19) ensures the non-negativity constraints of the service start time decision variables:

$$S_i^{nt} \geq 0, \forall\, i \in P, n \in N, t \in T \quad (19)$$

3. Implementation and computational results

3.1. Input parameters

The presented problem related to Home Health Care (HHC) for patients undergoing peritoneal Dialysis (PD), a service that has not yet been established in Tunisia. To validate the proposed mathematical model, random date are generated as input parameters.

In this paper, two distinct randomly generated instances are employed for computational tests. The proposed mathematical model is implemented using IBM ILOG CPLEX Optimization Studio 12.4 on a personal computer equipped with a Pentium (R) Dual-Core CPU at 2.00 GHz and 3 Go of RAM.

- **First tested instance:**

The first tested instance includes the following small-sized data:

```
Nurses=3;
Patients=2;
Shifts=1;
```

```
W=5;
TTij=[[0, 1, 1, 6, 1], [1, 0, 1, 4, 4], [1, 1, 0, 4, 4], [6, 4, 4, 0, 1], [1, 4, 4, 1, 0]];
Vit=[[1],[1]];
STjt=[[0],[0],[0],[3],[4]];
Apt=[[1],[1]];
Bpt=[[4],[3]];
```

- **Second tested instance:**

The second tested instance presents medium-sized data as follows:

```
Nurses=3;
Patients=5;
Shifts=7;
W=8;
TTij=[[0, 2, 8, 6, 1, 20,15,19],
[2, 0, 18, 16, 11, 2,5,9],
[8, 18, 0, 6, 1, 12,15,8],
[6, 16, 6, 0, 5, 20,15,19],
[1, 11, 1, 5, 0, 20,15,19],
[20, 2, 12, 20, 20, 0,5,20],
[15, 5, 15, 15, 15, 5,0,19],
[19, 9, 8, 19, 19, 20,19,0]];
Vit=[[1,1,1,1,1,1,1],[1,1,1,1,1,1,1],          [1,1,1,1,1,1,1],
[1,1,1,1,1,1,1], [1,1,1,1,1,1,1]];
STjt=[[0,0,0,0,0,0,0],[0,0,0,0,0,0,0],[0,0,0,0,0,0,0],[3,2,3,4
,5,6,7],[4,4,5,6,8,9,1],[9,8,5,7,5,6,7],[4,4,5,6,8,9,3],[6,4,5
,6,8,9,7]];
Apt=[[1,1,1,1,1,1,1],[1,1,1,1,1,1,1],          [1,1,1,1,1,1,1],
[1,1,1,1,1,1,1], [1,1,1,1,1,1,1]];
Bpt=[[200,200,200,200,200,200,200],[300,300,300,300,300,300,30
0],
[300,300,300,300,300,300,300],[300,300,300,300,300,300,300],
[300,300,300,300,300,300,300]];
```

3.2. Computational results

The outcomes from the initial set of small-scale computational test instances are detailed in Tables 3 and 4. As depicted in these tables, due to the high number of decision variables, we focus solely on the assigned variables, specifically those for which X_{ij}^{nt} is equal to 1.

The value of the objective function is approximately 169.

Table N°3: The assignment of patients to nurses and the nurses' routing (instance 1)

Nurse n	*Location i*	*Location j*	*Shift t*
1	1	5	1
1	1	5	2
1	1	5	3
1	1	5	4
1	1	5	5
1	1	5	6
1	1	5	7
1	5	1	1
1	5	1	2
1	5	1	3
1	5	1	4
1	5	1	5
1	5	1	6
1	5	1	7
2	2	4	4
2	2	4	5
2	2	4	6
3	3	4	1
3	3	4	2
3	3	4	3
3	3	4	7

Source: Authors

Table N°4: The obtained patients service start time at each shift (instance 1)

Patient i	*Nurse n*	*Shift t*	*Sint*
4	2	4	4
4	2	5	4
4	2	6	4
4	3	1	4
4	3	2	4
4	3	3	4
4	3	7	4
5	1	1	1
5	1	2	1
5	1	3	1
5	1	4	1
5	1	5	1
5	1	6	1
5	1	7	1

Source: Authors

We have also conducted a test with an instance involving 20 patients. After waiting for 2.09 hours, the execution process terminated with an error message of Out of Memory. Consequently, we decreased the number of patients to 10 but encountered the same Out of Memory error after 38.14 minutes of execution. We further reduced the number of patients to 5.

Tables 5 and 6 display the results from the medium-sized instances that were generated, along with the respective execution time.

The optimal solution obtained was 730.

Table N°5: The assignment of patients to nurses and the nurses' routing (instance 2)

Nurse n	*Location i*	*Location j*	*Shift t*
1	1	5	1
1	1	5	2
1	1	5	3
1	1	5	4
1	1	5	5
1	1	5	6
1	1	5	7
1	4	1	1
1	4	1	2
1	4	1	3
1	4	1	4
1	4	1	5
1	4	1	6
1	4	1	7
1	5	4	1
1	5	4	2
1	5	4	3
1	5	4	4
1	5	4	5
1	5	4	6
1	5	4	7
2	2	6	1
2	2	6	2
2	2	6	3
2	2	6	4
2	2	6	5
2	2	6	6

2	2	6	7
2	6	7	1
2	6	7	2
2	6	7	3
2	6	7	4
2	6	7	5
2	6	7	6
2	6	7	7
2	7	2	1
2	7	2	2
2	7	2	3
2	7	2	4
2	7	2	5
2	7	2	6
2	7	2	7
3	3	8	1
3	3	8	2
3	3	8	3
3	3	8	4
3	3	8	5
3	3	8	6
3	3	8	7
3	8	3	1
3	8	3	2
3	8	3	3
3	8	3	4
3	8	3	5
3	8	3	6
3	8	3	7

Source: Authors

Table N°6: The obtained patients Service start time at each shift (instance 2)

Patient i	*Nurse n*	*Shift t*	*Sint*
4	1	1	10
4	1	2	10
4	1	3	11
4	1	4	12
4	1	5	14
4	1	6	15
4	1	7	7
5	1	1	1
5	1	2	1
5	1	3	1
5	1	4	1
5	1	5	1
5	1	6	1
5	1	7	1
6	2	1	2
6	2	2	2
6	2	3	2
6	2	4	2
6	2	5	2
6	2	6	2
6	2	7	2
7	2	1	16
7	2	2	15
7	2	3	12
7	2	4	14
7	2	5	12
7	2	6	13

7	2	7	14
8	3	1	8
8	3	2	8
8	3	3	8
8	3	4	8
8	3	5	8
8	3	6	8
8	3	7	8

Source: Authors

We have determined that the exact method presents the optimal solution but it is both time and memory consuming, especially for real world instances. For a future study, we plan to address large-scale instances of the problem by employing Machine Learning-based approaches to enhance efficiency and scalability.

Conclusion

The situation of patients with kidney failure is critical. To ameliorate the quality of healthcare in Tunisia and improve the patients' satisfaction, we propose to require to HHC for peritoneal dialysis.

In this paper, we proposed a mathematical model designed to optimize patients scheduling for peritoneal Dialysis with the context of Home Health Care. The objective was to minimize the total patients' service start time. We have incorporated two novel constraints into our model: additional nurses visits for cycler maintenance and patients analyzes.

To the best of our knowledge, this research marks the pioneering effort to integrate HHC for peritoneal Dialysis in Tunisia. The proposed methodology is currently being evaluated by the Tunisian healthcare authorities. Future research endeavors should focus on conducting extensive tests for large-scale instances to further validate the model's applicability and effectiveness.

Our contribution is as follows: (1) development of a novel mathematical model for patient scheduling in HHC for peritoneal dialysis, (2) introduction of two innovative constraints to enhance the model's practicality and (3) pioneering exploration of HHC integration for peritoneal dialysis in Tunisia.

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